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## Integral resonant damping for high-bandwidth control of piezoceramic stack actuators with asymmetric hysteresis nonlinearity

Guo-Ying Gu<sup>a</sup>, Li-Min Zhu<sup>a,\*</sup>, Chun-Yi Su<sup>b</sup><sup>a</sup> State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China<sup>b</sup> Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Quebec H3G 1M8, Canada

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### ABSTRACT

This paper presents a novel control scheme for high-bandwidth control of piezoceramic stack actuators (PSAs). For this purpose, we first characterize and compensate for the asymmetric hysteresis nonlinearity of the PSA. A linear integral resonant controller is then designed as a means for damping the resonant modes of the dynamic system with the hysteresis compensation. Finally, a tracking controller and feed-forward input are developed to minimize the tracking errors and improve the closed-loop tracking bandwidth. To verify the effectiveness and efficiency of the proposed control scheme, a PSA-actuated positioning platform is built and comparative experiments are conducted. Experimental results demonstrate that the proposed controller achieves robust broadband nanopositioning of the PSA by improving the tracking bandwidth from 22 Hz (with an integral controller) to 657 Hz.

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### 1. Introduction

Along with the rapid development of nanoscience and nanotechnology, nanopositioning stages are widely applied in many precision manufacturing equipments such as scanning probe microscopes (SPMs) [1,2] and diamond turning machines [3]. For instance, in SPMs, a nanopositioning stage is utilized to scan the probe over a sample surface with the nanometer or sub-nanometer positioning resolution and to control the interaction between the probe and the sample surface. Besides high-precision positioning, high-bandwidth operation of the stage is imperative when imaging the sample with a fast time-varying nature in order to avoid distortions.

A pivotal component of nanopositioning stages is a piezoceramic actuator due to its excellent advantages of high displacement resolution, fast response time and high stiffness [1,4]. However, the piezoceramic actuators have two main drawbacks limiting the accuracy and speed of the nanopositioning stages: (i) inherent hysteresis nonlinearity in the piezoceramic material [4,5] and (ii) low damped resonant modes due to the mechanical dynamics [6,7].

Hysteresis nonlinearity in the piezoceramic materials is one of the main problems degrading the positioning accuracy. In nanopositioning stages actuated by piezoceramic actuators, the existence of the hysteresis nonlinearity can produce positioning error up to

15% of the moving range at operating frequencies well below the first resonance mode of the piezo-actuated stage [8]. In worst case, it may cause the oscillation, or even instability of the control systems [4]. To remedy the hysteresis, many efforts have been made in the literature. The most common approach is to construct a model that can accurately describe the hysteresis nonlinearity and then preshape the input voltage based on the inverse of the model [9–13]. Rather than constructing the inverse hysteresis model, some attempts have been made to apply the feedback control approaches [14–16] to mitigate the hysteresis nonlinearity. As an alternative, charge actuation [17] of piezoceramic actuators has shown to be an effective method significantly reducing hysteresis nonlinearity compared with generally used voltage actuation. However, the charge amplifier has not been widely adopted due to its implementation complexity and cost.

Resonant modes are the major issue hindering the operation speed of the piezo-actuated nanopositioning stages, which restricts operating frequencies to less than 1/10–1/100th of the first resonance frequency of the stages to avoid the positioning problem at high operating speeds [18]. Although one may build a sufficiently stiff and small piezo-actuated stage to increase the operation speed, its maximum travel range is limited to a few microns. Even in this case, the tracking bandwidth is still limited by the resonant modes of the actuators [19]. Therefore, it requires development of control techniques to suppress the vibration effect. To address this issue, several strategies have been developed in the literature, including the input-shaping technique [20,21], the notch filter [6,22], and the inversion-based approach [23]. However, the solutions using these techniques are sensitive to variations of the

\* Corresponding author. Tel.: +86 2134206545.

E-mail addresses: [guguaying@sjtu.edu.cn](mailto:guguaying@sjtu.edu.cn) (G.-Y. Gu), [zhulm@sjtu.edu.cn](mailto:zhulm@sjtu.edu.cn) (L.-M. Zhu), [cysu@alcor.concordia.ca](mailto:cysu@alcor.concordia.ca) (C.-Y. Su).

resonant mode parameters in the system model [24,25]. As an alternative, feedback control is introduced to suppress the resonant vibrations, such as positive position feedback control [26], resonant control [27] and integral resonant control (IRC) [28]. Recently, the IRC strategy is demonstrated as a simple means for damping the vibrations, which has been successfully applied to piezoceramic tube actuators for fast nanoscale positioning [25]. However, the charge control is used in these applications. The use of IRC strategy for piezoceramic stack actuators driven by voltage control is still missing.

1.1. Contribution of this paper

The contribution of this paper is to propose a combined control scheme for high-bandwidth control of piezoceramic stack actuators (PSAs). The control objective is achieved by the following steps: (1) an inverse hysteresis compensator is designed to mitigate the asymmetric hysteresis nonlinearity using a modified Prandtl-Ishlinskii (MPI) model; (2) the IRC strategy is introduced as a means for damping the dynamic system with the inverse hysteresis compensation; (3) an integral tracking controller is developed to minimize tracking errors of the desired position  $y_d$  and the actual position  $y$ ; (4) a feedforward input with the inverse dc gain of the system is utilized to further improve the tracking bandwidth of the closed-loop system; and (5) comparative experiments on a PSA-actuated stage are conducted in real time to verify the effectiveness of the proposed control scheme.

1.2. Outline

The remainder of this paper is organized as follows. In the next section, the experimental setup is presented. In Section 3, the proposed control scheme including an inverse hysteresis compensator, an integral resonant controller, a feedback tracking controller and a feedforward input is developed. After that, comparative experiments and evaluations are performed in Section 4, followed by the conclusion in Section 5.

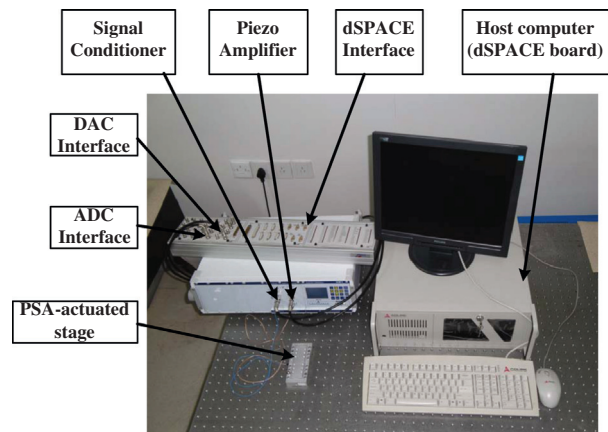
2. Experimental setup

A flexure-guided nanopositioning stage is used in this paper to implement and test the proposed control scheme. The stage is actuated by a preloaded piezoceramic stack actuator (PSA) from Piezomechanik in Germany. A power amplifier with a gain of 15 is used to drive the PSA. A high-resolution strain gauge sensor (SGS) integrated into the PSA is adopted to measure the real-time position. A signal conditioner is used to convert the measured position to analog voltage in the range of 0–10 V. A dSPACE DS1103 board equipped with the 16-bit DAC and 16-bit ADC is utilized to output the excitation for the power amplifier and capture the real-time displacement information from the signal conditioner. As an illustration, the experimental platform is shown in Fig. 1a.

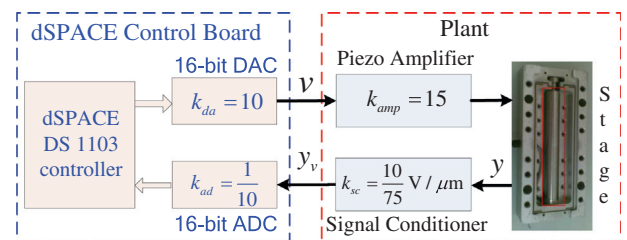
For modeling and control purpose, the PSA-actuated nanopositioning system is treated as a signal-input-signal-output system. The input is the voltage  $v$  applied to the piezo amplifier and the output is the corresponding displacement  $y$  from the SGS. The block diagram of the entire experimental setup is shown in Fig. 1b. In the following development, the proposed control scheme will be addressed in detail.

3. Controller design

The control objective in high-bandwidth nanopositioning is to not only minimize the tracking error but also maximize the tracking speed of the PSA. As the actuator is inherently nonlinear (i.e.



(a) Experimental platform



(b) Block diagram

Fig. 1. The experimental setup.

the hysteresis nonlinearity) and lightly damped, it requires the multi-algorithm fusion control strategy in the control loop. In this paper, an inverse hysteresis compensator is firstly adopted to cancel the hysteresis nonlinearity. Then, an integral resonant controller is utilized to damp the hysteresis compensated system's first resonant mode. Finally, an integral controller and a feedforward input are implemented to achieve high-bandwidth nanopositioning of the PSA.

3.1. Hysteresis compensator

In this section, we characterize and compensate for the hysteresis nonlinearity of the PSA. The compensation is necessary in order to linearize the system and therefore to make easy the synthesis of a linear integral resonant controller for vibration damping in the following development.

The first step in constructing the inverse hysteresis compensator is to characterize the hysteresis nonlinearity in the PSA with an effective hysteresis model. The hysteresis of the PSA is an inherent multi-valued nonlinearity with the asymmetric characteristic, which can be observed in the relationship between the applied voltage and corresponding displacement of the PSA shown in Fig. 2. To describe this asymmetric hysteresis, the modified Prandtl-Ishlinskii (MPI) model [29] defined in terms of weighted play operators and a polynomial input function is utilized in this work

$$y(t) = P_g[v](t) = g(v(t)) + \int_0^R p(r)F_r[v](t)dr \tag{1}$$

where  $v(t)$  is the input,  $y(t)$  is the output,  $g(v(t)) = a_1 v^3(t) + a_2 v(t)$  is a polynomial input function with constant  $a_1$  and  $a_2$ ,  $p(r)$  is a density function that is generally calculated from the experimental data, and  $F_r[v](t)$  is a one-side play operator expressed as

$$\begin{aligned} F_r[v](0) &= f_r(v(0), 0) \\ F_r[v](t) &= f_r(v(t), F_r[v](t_i)) \end{aligned} \tag{2}$$

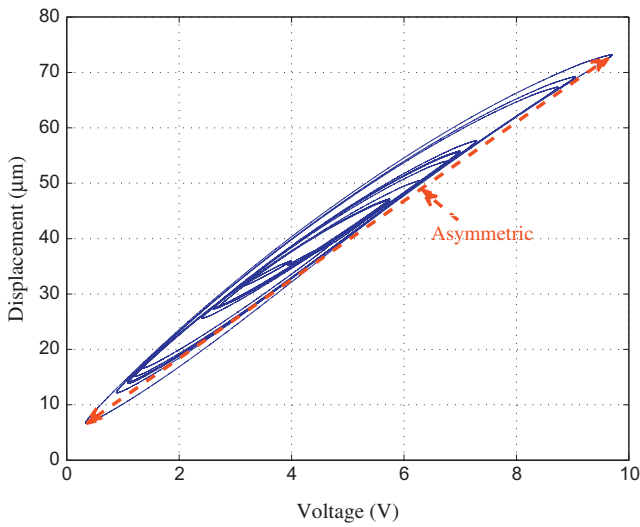


Fig. 2. The hysteresis nonlinearity of the PSA.

for  $t_i < t \leq t_{i+1}, 0 \leq i \leq N - 1$  with

$$f_r(v, w) = \max(v - r, \min(v, w)) \quad (3)$$

where  $0 = t_0 < t_1 < \dots < t_N = t_E$  is a partition of  $[0, t_E]$ , such that the function  $v(t)$  is monotone on each of the subintervals  $[t_i, t_{i+1}]$ . The argument of the operator  $F_r[v](t)$  is written in square brackets to indicate the functional dependence, since it maps a function to another function. The reader may refer to [29] for detailed descriptions and discussions of the MPI model.

To compensate for a hysteresis nonlinearity that has been modeled with a hysteresis model  $\Gamma$ , the common idea is to construct the inverse hysteresis model  $\Gamma^{-1}$ , which is put in series with the real hysteretic system as shown in Fig. 3. In order to conveniently implement the real-time inverse hysteresis compensator, the definition of MPI model given in (1) can be approximated in the discrete form with a finite number of the play operators as follows

$$y_g(t) = P_g[v](t) = a_1 v^3(t) + a_2 v(t) + \sum_{i=1}^n b(r_i) F_{r_i}[v](t) \quad (4)$$

where  $n$  is the number of the adopted play operators for modeling, and  $b(r_i)$  is the weighted constant for the threshold  $r_i$ . In this work, to describe the asymmetric hysteresis of the tested PSA, the value of  $n$  is chosen as 10, and the threshold values  $r_i$  in the MPI model (4) are chosen as

$$r_i = \frac{i-1}{n} \|v(t)\|_{\infty}, \quad i = 1, 2, \dots, n \quad (5)$$

with  $\|v(t)\|_{\infty} = 1$  in the normalized case. The weighted parameters  $b(r_i)$ , and coefficients  $a_1$  and  $a_2$  of polynomial input function in (4) are identified by the particle swarm optimization algorithm [29], which are listed in Table 1. As shown in Fig. 4, the hysteresis loops generated by the identified MPI model are also compared with the experimental results, which clearly verifies the effectiveness of the model.

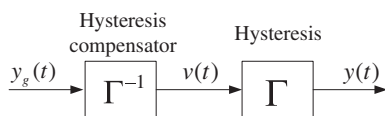


Fig. 3. Block representation for inverse hysteresis compensation.

Table 1  
Identified parameters of the MPI model.

Number	$r_i$	$b_i$	$a_i$
1	0	0.2313	-0.1569
2	0.1	0.3059	0.4603
3	0.2	0.0155	
4	0.3	0.0752	
5	0.4	0.0683	
6	0.5	0.0252	
7	0.6	0.0035	
8	0.7	0.0094	
9	0.8	0.0264	
10	0.9	0.0032	

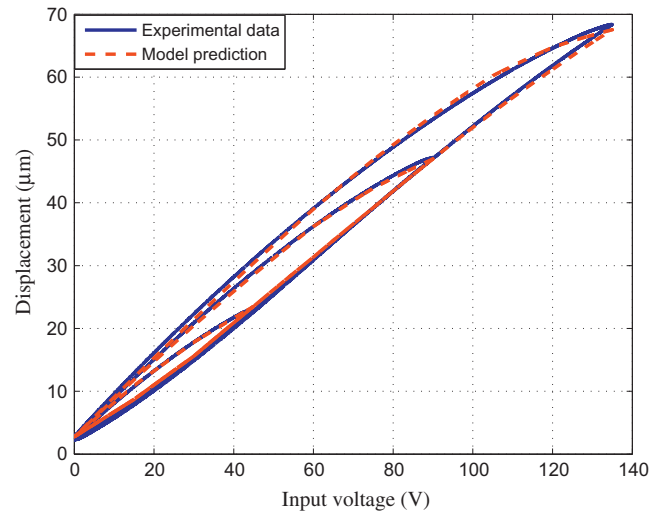


Fig. 4. Comparison of the hysteresis loops generated by the PSA and the identified MPI model.

On the basis of the inverse model of the classical Prandtl-Ishlinskii (CPI) model [9] and inspired by the work in [30], the MPI model (4) is re-expressed as the following superposition [29]

$$y_g(t) = P_g[v](t) = P_c[v](t) + P[v](t) \quad (6)$$

where  $P_c[v](t) = a_1 v^3(t)$  is a new model component, and  $P[v](t) = a_2 v(t) + \sum_{i=1}^n b(r_i) F_{r_i}[v](t)$  is by nature the CPI model [9].

The synthesis of efficient real-time control algorithms for the compensation of the asymmetric hysteresis nonlinearity represented by the MPI model is based on the implicit operator equation

$$v(t) = P^{-1}[w](t) \quad (7)$$

for the inverse compensator

$$v(t) = P_g^{-1}[y_g](t) \quad (8)$$

where  $w = y_g - P_c[v]$ , and  $P^{-1}$  is the inverse of the CPI model expressed as

$$P^{-1}[w](t) = \hat{a}_2 w(t) + \sum_{j=1}^n \hat{b}_j F_{\hat{r}_j}[w](t) \quad (9)$$

with

$$\begin{aligned} \hat{r}_j &= a_2 r_j + \sum_{i=1}^{j-1} b_i (r_j - r_i) \\ \hat{a}_2 &= \frac{1}{a_2} \\ \hat{b}_j &= -\frac{b_j}{(a_2 + \sum_{i=1}^{j-1} b_i)(a_2 + \sum_{i=1}^j b_i)} \end{aligned} \quad (10)$$

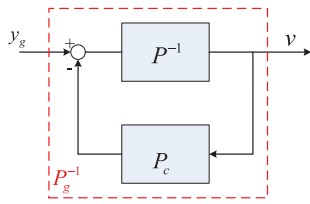


Fig. 5. Signal flow chart of the inverse compensator  $P_g^{-1}$ .

By this way, the analytical inverse hysteresis model of the MPI model can be obtained. As an illustration, Fig. 5 shows the signal flow chart of the inverse compensator  $P_g^{-1}$  for the MPI model. The MPI inverse compensator has been implemented for hysteresis compensation. Fig. 6 shows the experimental results with and without the inverse compensator and demonstrates that the hysteresis has been reduced to 3.3% instead of about 13.6% ( $= \frac{h}{H} \times 100\%$ ) without the inverse compensation. It can be seen that the hysteresis nonlinearity is greatly mitigated with the inverse hysteresis compensator and the resulted relationship between the desired position and the actual position is almost linear and symmetric. In the following development, the integral resonant controller can be designed for the hysteresis compensated system to attenuate its vibration effect.

### 3.2. Damping controller

In this section, the integral resonant control (IRC) strategy is applied as a means for damping the dynamic system with the hysteresis compensation. For the purpose of IRC design and analysis, a model of the linear dynamic system is required [25,28]. In Section 3.1, the inverse hysteresis compensator is introduced to mitigate the hysteresis linearity, which makes it possible to identify the linearized dynamic system (i.e. the PSA-actuated stage with the inverse hysteresis compensation) for the IRC design.

As reported in our previous work [4], the PSA-actuated stage can be modeled as a third-order linear dynamic model preceded by an input hysteresis nonlinearity. In Section 3.1, the hysteresis nonlinearity has been compensated by the IHC. Therefore, the dynamic model of the hysteresis compensated system can be represented as a third-order structure. To identify the model, a bandlimited white noise signal is used to excite the hysteresis

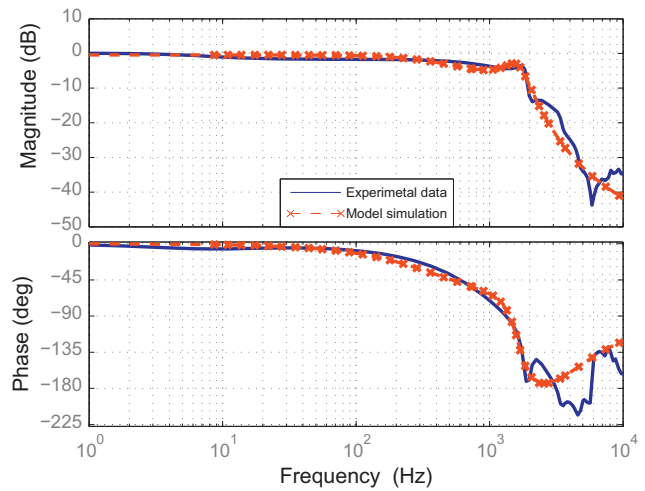


Fig. 7. Comparison of frequency responses of experimental results and model simulation results.

compensated system. The dSPACE control system is utilized to simultaneously capture the excitation voltage and the corresponding measured displacement. Then, the system identification toolbox of MATLAB can be adopted to obtain the identified dynamic model represented as

$$G(s) = \frac{Y(s)}{Y_g(s)} = \frac{470.4068(s^2 + 4.067e004s + 5.882e008)}{(s + 2726)(s^2 + 4030s + 1.067e008)}. \quad (11)$$

Fig. 7 shows the comparison of frequency responses of experimental results and model simulation results to demonstrate the effectiveness of the identified model. It can be seen that the identified model captures the dynamics of the system with sufficient accuracy.

The principle of the IRC strategy [25,28] is represented as the block diagram in Fig. 8a. It consists of a linear collocated dynamic system  $G_2$ , a constant artificial feed-through term  $D_f$  with  $D_f < 0$ , and an integral controller  $C$ . The objective of the IRC strategy is to design the artificial feed-through term  $D_f$  and the integral controller  $C$  such that the resonant modes of the system  $G_2$  are damped. For simplicity, an equivalent block diagram of the IRC strategy [25] is also shown in Fig. 8b. It should be mentioned that

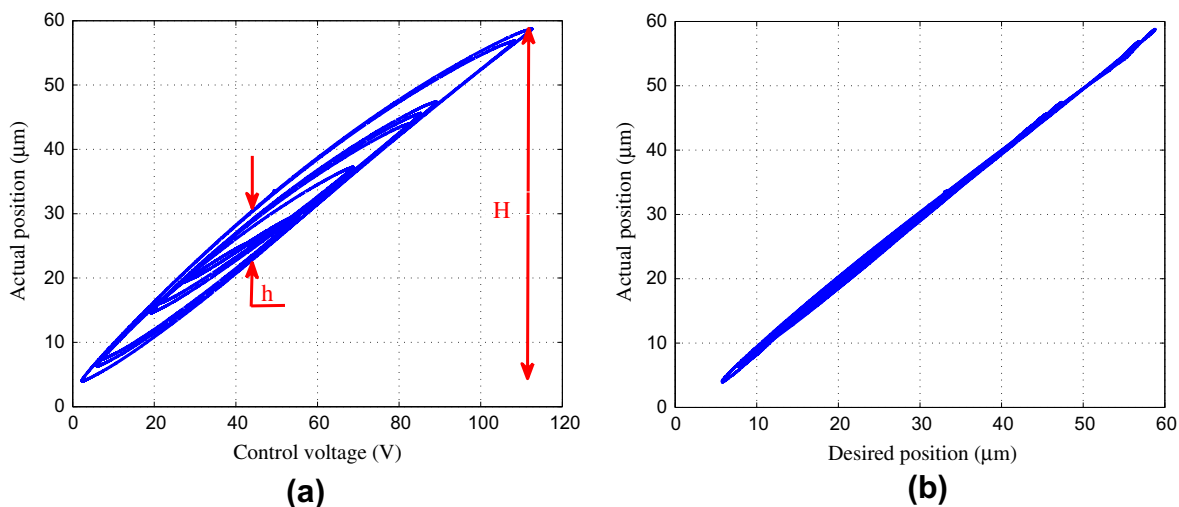


Fig. 6. (a) Curves showing the hysteresis of the PSA in open loop control. (b) Curves showing the measured position versus the desired input using the inverse hysteresis compensator. Experimental results with and without the inverse hysteresis compensator.

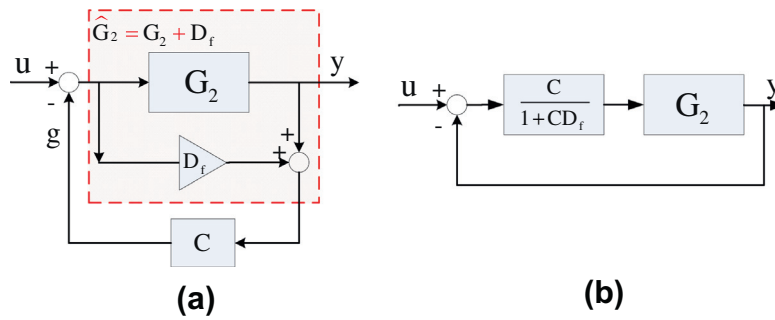


Fig. 8. (a) Schematic diagram of the IRC strategy. (b) Equivalent schematic diagram of the IRC strategy.

in the IRC strategy, the dynamic system  $G_2$  is generally a two-order transfer function with the form of  $\frac{K(s^2+2\xi_1\omega_1s+\omega_1^2)}{s^2+2\xi_2\omega_2s+\omega_2^2}$  [25]. By applying the IRC strategy for the system (11) in this work, the model (11) should be re-expressed as

$$G(s) = G_1(s) \times G_2(s). \tag{12}$$

with

$$\begin{cases} G_1(s) = \frac{2726}{s+2726} \\ G_2(s) = \frac{0.1726s^2+7020s+1.015e008}{s^2+4030s+1.067e008} \end{cases} \tag{13}$$

The first step in applying the IRC strategy is to determine the artificial feed-through term  $D_f$ . As shown in Fig. 8a, by introducing the feed-through term  $D_f$ , an augmented system  $\hat{G}_2 = G_2 + D_f$  is formed. It can be seen that the augmented system  $\hat{G}_2$  and the original system  $G_2$  have the same poles. By selecting a sufficient  $D_f$ , a pair of zeros below the first resonant mode is introduced to guarantee zero-pole interlacing for higher frequency modes [25,28].

For the model  $G_2$  described in (13), using the root-locus technique [25], a feed-through term of  $D_f = -1.8$  is chosen to introduce a pair of zeros below the first resonance mode. The frequency responses of the open-loop system  $G_2$  and the augmented system  $G_2 + D_f$ , where  $D_f = -1.8$ , are plotted in Fig. 9. It can be seen that the feed-through term  $D_f$  introduces a pair of complex zeros at the frequency of 1.19 kHz to provide a phase lead. It should be noted that due to the negative sign in  $\hat{G}_2$  additional  $-180^\circ$  makes

the phase response of the modified transfer function lie between  $-180^\circ$  and  $0^\circ$  as shown in Fig. 9.

With the modified transfer function  $\hat{G}_2$ , the augmented transfer function  $G_m$  for (12) is obtained as  $G_m = G_1 \times \hat{G}_2$ . Fig. 9b also shows the frequency response for the augmented transfer function  $G_m$ . It is observed that the phase of  $G_m$  is bounded between  $-270^\circ$  and  $-90^\circ$  due to the introduction of the feed-through term  $D_f$ . Therefore, a simple negative integral

$$C(s) = \frac{-k}{s}, \quad \text{with } k > 0 \tag{14}$$

can be applied to provide a  $90^\circ$  phase lead. To evaluate the stability of such a controller, the loop gain of  $CG_m$  is examined. Thus, the phase of  $CG_m$  is equal to the phase of  $G_m$  plus  $90^\circ$ . The resulting phase response will be within  $-180^\circ$  and  $0^\circ$ , which produces excellent phase margin and infinite gain margin. Using the root-locus technique [25,28], the controller gain  $k$  is chosen as 754 to maximize damping. In such a case, Fig. 10 shows the frequency response of the open-loop transfer function for verification.

*Remark:* It should be noted that the IRC is designed based on the linear control theory using a transfer function method, which cannot be directly applied to handle the non-smooth hysteresis non-linearity of the PSA-actuated system. In this work, the IHC is introduced in advance to remedy the hysteresis nonlinearity. As a result, the PSA-actuated system with the IHC is by nature treated as a linear dynamic system, which makes it possible to develop the IRC. Therefore, the IHC part developed in Section 3.1 is a necessary control unit for the IRC design in our proposed control scheme.

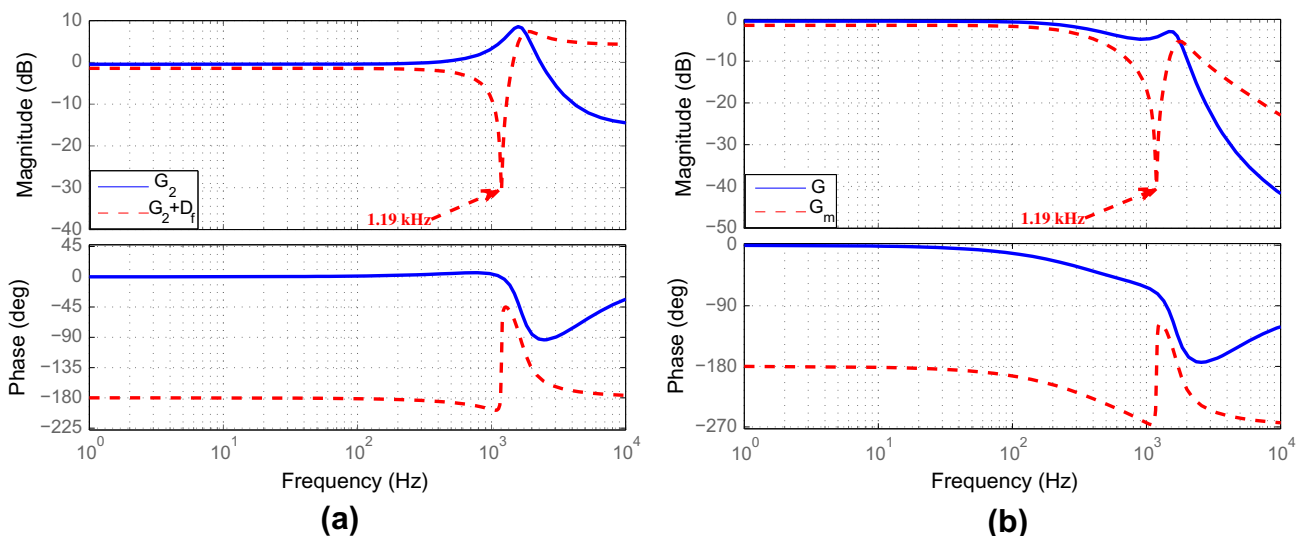


Fig. 9. (a)  $G_2$  and  $G_2 + D_f$ . (b)  $G$  and  $G_m$  Frequency responses to describe the effect of the feed-through term  $D_f$  on the transfer functions  $G_2$  and  $G$ .

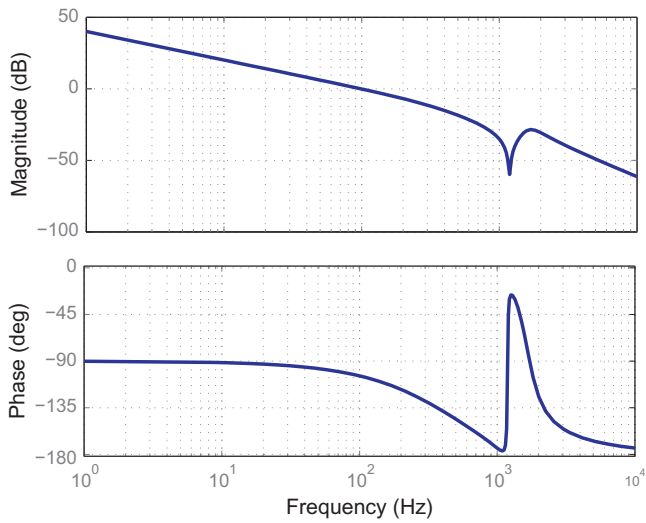


Fig. 10. Frequency responses of the open-loop transfer function  $CG_m$  with  $k = 754$ .

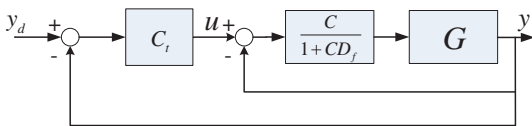


Fig. 11. Block of the tracking control system with the IRC controller and tracking controller.

### 3.3. Tracking controller

The foremost control objective in nanopositioning is to minimize tracking errors between the desired position  $y_d$  and the actual position  $y$ . In the aforementioned sections, we have addressed related compensation methods for the hysteresis and vibration effects. In this section, a secondary feedback tracking controller  $C_t$  is designed to compensate for disturbances and modeling uncertainties as shown in Fig. 11.

As addressed in Section 3.2, the IRC strategy has been utilized to overcome the challenge of the low gain margin problem [7,31] in the PSA. It thus enables to design a high-gain tracking controller for the system after the IRC compensation. Due to the inverting behavior of the IRC loop [25], a proportional-integral controller  $C_t$  with a negative sign is developed in this work, whose transfer function is expressed as

$$C_t(s) = -\frac{k_p s + k_i}{s} \quad (15)$$

where  $k_p > 0$  is the proportional gain and  $k_i > 0$  is the integral gain.

Therefore, the transfer function of the closed-loop system is expressed as

$$\frac{Y(s)}{Y_d(s)} = \frac{C_t(s)C_1(s)G(s)}{1 + C_1(s)(1 + C_t(s))G(s)} \quad (16)$$

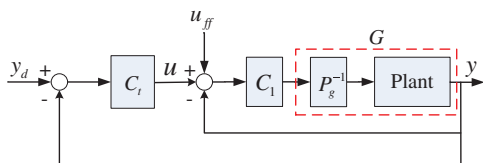


Fig. 12. Block diagram of the proposed control scheme with the inverse hysteresis compensator  $P_g^{-1}$ , the IRC controller  $C_1$ , the tracking controller  $C_t$  and the feedforward input  $u_{ff}$ .

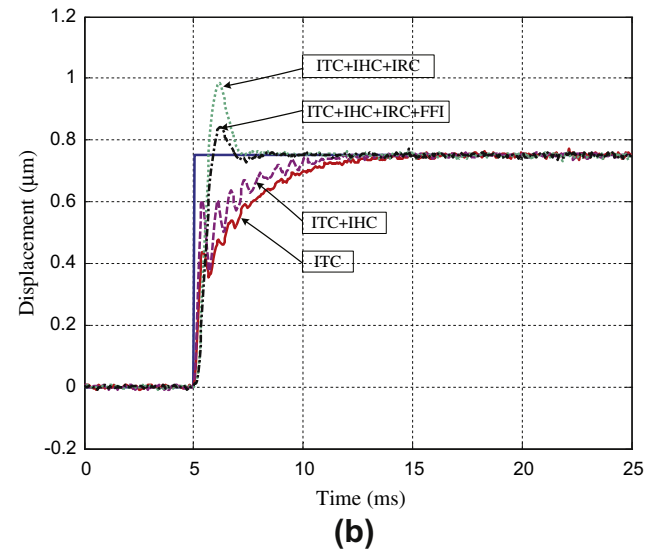
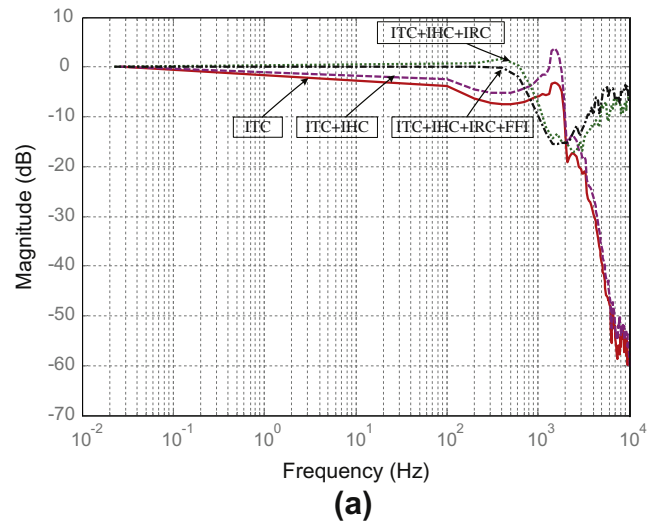


Fig. 13. Control performance comparison of the four developed control strategies. (a) Frequency responses. (b) Step responses.

Table 2  
Control parameters and performances of the four kinds of controllers.

Controller	ITC	ITC + IHC	ITC + IHC + IRC	ITC + IHC + IRC + FFI
$C_t$	$\frac{s+1000}{s}$	$\frac{s+1000}{s}$	$-\frac{4s+3500}{s}$	$-\frac{4s+3500}{s}$
$P_g^{-1}$	0	Eq. (8)	Eq. (8)	Eq. (8)
$C_1$	0	0	$-\frac{754}{s+1357}$	$-\frac{754}{s+1357}$
$u_{ff}$	0	0	0	0.8922
Bandwidth (Hz)	22	118	779	657
Setting time (ms)	9	7	4	3
Vibration	Large	Large	Small	Small
Overshoot	Small	Small	Large	Small

with

$$C_1(s) = \frac{C(s)}{1 + C(s)D_f} \quad (17)$$

### 3.4. Feedforward controller

Feedforward control is an effective method to improve the tracking bandwidth of a closed-loop system by bypassing the

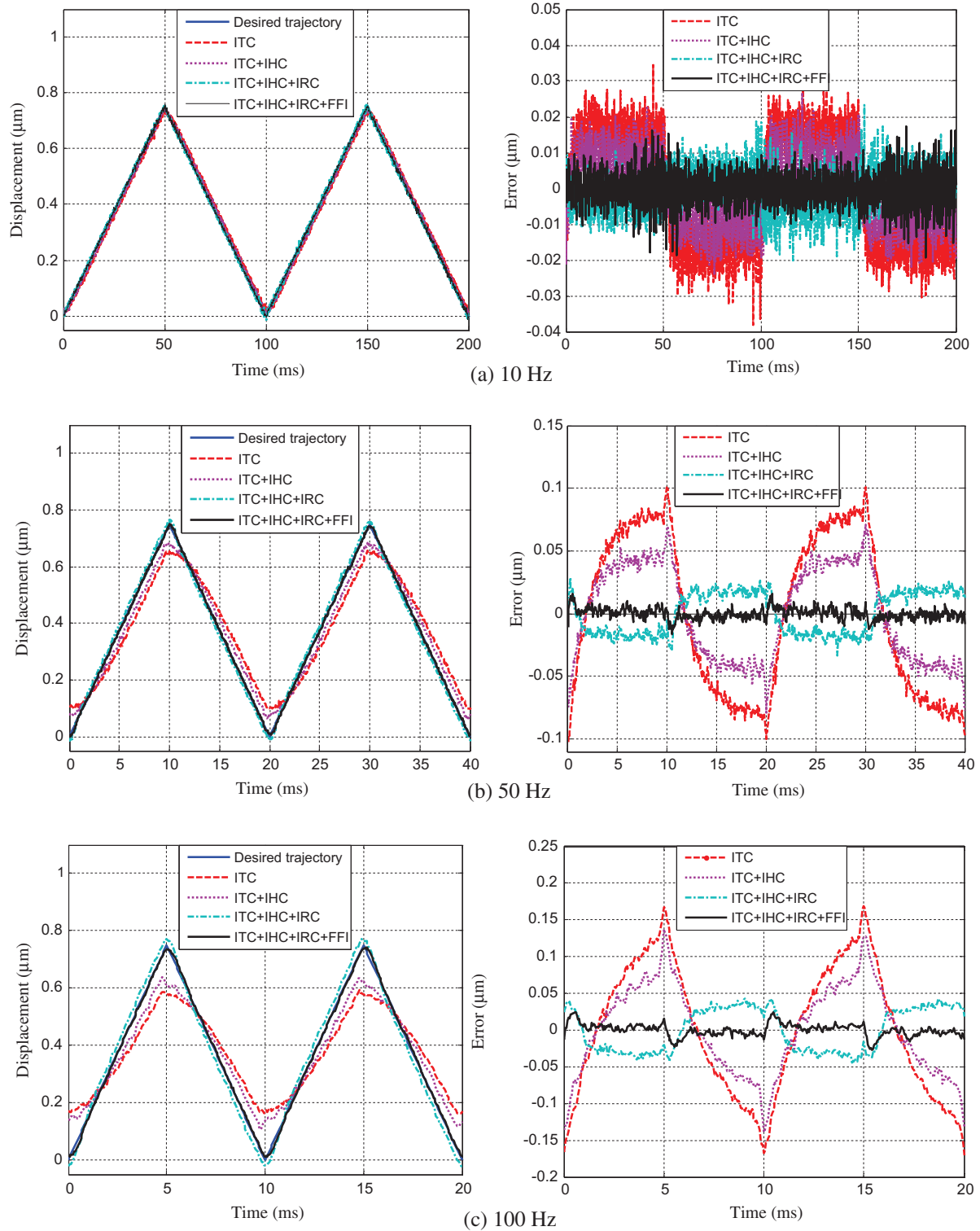


Fig. 14. Comparison of the tracking performances of the different controllers for triangular trajectories under various input frequencies.

tracking controller or inverting dynamics [18,25,32,33]. Similar to the reported work in [25], the inverse dc gain of the system is simply used to provide the performance improvement in this application and we will verify this further in the following experiments.

In this work, the feedforward input is denoted as  $u_{ff}$ , which is generated by the reference input and the inverse dc gain of the damped system as follows

$$u_{ff} = y_d \left( \left. \frac{C_1(s)G(s)}{1 + C_1(s)G(s)} \right|_{s=0} \right)^{-1}. \quad (18)$$

Therefore, the block diagram of the proposed control scheme with the inverse hysteresis compensator  $P_g^{-1}$ , the IRC controller  $C_1$ , the tracking controller  $C_t$  and the feedforward input  $u_{ff}$  is schematically shown in Fig. 12.



#### 4. Experimental implementation

In this section, the proposed control scheme is implemented on the PSA-actuated stage described in Section 2. For ease of comparison, four controllers are designed: (1) the integral tracking controller (ITC); (2) the integral tracking controller with the inverse hysteresis compensator (ITC + IHC); (3) the integral tracking controller with the inverse hysteresis compensator and the IRC damping (ITC + IHC + IRC); and (4) the integral tracking controller with the inverse hysteresis compensator, the IRC damping and the feedforward input (ITC + IHC + IRC + FFI).

(1) ITC: It is well known that higher control gains for  $k_p$  and  $k_i$  commonly contribute to smaller tracking errors and faster trajectory convergence. However, doing so may compromise the stability of the system. Therefore, the values of  $k_p$  and  $k_i$  cannot be arbitrarily set, and a tradeoff between stability and performance need to be experimentally established when tuning the parameter values. In this work, the ITC was designed to maximize tracking bandwidth. The maximum gain was restricted to  $k_p = 1$  and  $k_i = 1000$  by the gain-margin of only 2.9 dB. Fig. 13a shows the experimental frequency response of the system with the ITC. It can be seen that the control bandwidth (i.e.  $-3$  dB bandwidth) of the ITC is 22 Hz. Fig. 13b shows the step response of the system with the ITC. We observe that the output exhibits oscillations and converges slowly with a steady state error of roughly 15 nm after about 9 ms.

(2) ITC + IHC: By adding an asymmetric inverse hysteresis compensator to the ITC as addressed in Section 3.1, the bandwidth can be extended to 118 Hz at the expense of gain-margin as shown in Fig. 13a. From the step response shown in Fig. 13b, the output converges more quickly than in the case of the ITC, while it exhibits more undesirable oscillations due to the lightly damped resonance mode.

(3) ITC + IHC + IRC: Following the procedure in Section 3.2, the IRC damping controller was further designed for the system to tackle the low gain margin problem of the resonant modes in the PSA-actuated system. It should be mentioned that it is the IRC that make the larger control gains of  $k_p$  and  $k_i$  possible, which is the motivation of this work to develop the IRC for high-bandwidth control of the PSA. Thereafter, a high-gain ITC with  $k_p = 4$  and  $k_i = 3500$  was developed while maintaining the gain-margin of 13 dB. In this case, as shown in Fig. 13a, the control bandwidth is extended to 779 Hz. However, some of this bandwidth is uncontrolled. The faster response can also be demonstrated by the step response shown in Fig. 13b, where a large overshoot is observed.

(4) ITC + IHC + IRC + FFI: To further improve the control performance, the FFI as addressed in Section 3.4 was finally introduced in the developed controller. As shown in Fig. 13, the effect of the improved bandwidth is demonstrated by both the frequency response and step response. Compared with the results of the ITC + IHC + IRC controller, the overshoot is significantly reduced, while a higher control bandwidth is possessed. In summary, Table 2 lists the control parameters and performances of the four kinds of controllers for comparison.

As a last example to evaluate the tracking performance, Fig. 14 shows the experimental results of the closed-loop system with different controllers for triangular trajectories under various input

frequencies. In addition, the RMS tracking errors are compared in Table 3. From the figure and table, it can be demonstrated that the high-speed and high-precision performances of the proposed ITC + IHC + IRC + FFI controller is significantly better than the others.

#### 5. Conclusion

In this paper, we present a novel control scheme for high-precision and high-bandwidth control of the PSA. The control scheme first deals with the asymmetric hysteresis nonlinearity by an inverse hysteresis compensator. As a result, the linear IRC strategy can be applied as a means for damping the resonant modes of the dynamic system with the inverse hysteresis compensation. Finally, the tracking controller and the feedforward input are designed to achieve the fast nanopositioning of the PSA. We have demonstrated the effectiveness and efficiency of the proposed method when applied to a PSA-actuated positioning stage. The comparative experimental results confirm that the proposed controller successfully increases the closed-loop tracking bandwidth of the traditional integral tracking controller from 22 Hz to 657 Hz.

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**Table 3**  
RMS tracking errors for triangular trajectories under various input frequencies.

Controller	10 Hz	50 Hz	100 Hz
ITC	16.0 nm	63.1 nm	90.4 nm
ITC + IHC	9.6 nm	38.4 nm	62.4 nm
ITC + IHC + IRC	5.5 nm	16.8 nm	29.7 nm
ITC + IHC + IRC + FFI	4.0 nm	4.6 nm	8.6 nm

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