Sensors and Actuators A 165 (2011) 303–309

Contents lists available at ScienceDirect

Sensors and Actuators A: Physical

journal homepage: www.elsevier.com/locate/sna

# Modeling of rate-dependent hysteresis in piezoelectric actuators using a family of ellipses

GuoYing Gu, LiMin Zhu*

*State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China*

ARTICLE INFO

*Article history:*
Received 16 March 2010
Received in revised form 31 August 2010
Accepted 28 September 2010
Available online 8 October 2010

*Keywords:*
Rate-dependent hysteresis
Piezoelectric actuator
Modeling
Ellipse

ABSTRACT

In this paper, a new ellipse-like mathematic model is proposed to describe the rate-dependent hysteresis in piezoelectric actuators. Since the expressions of the model are completely analytical and can be determined only by a set of parameters, this method simplifies the modeling of complicated hysteresis behaviors. To represent the hysteresis effects, experiments are performed with designed sinusoidal excitations under different frequencies in the range 0.5–300 Hz. The rate-dependent hysteresis is characterized as increasing maximum hysteresis error (MHE) and decreasing peak-to-peak output amplitude (PPOA) phenomenons with the increase of input frequencies. Then, the parameters of the developed model are extracted from the experimental data using the direct least square method through MATLAB offline. The simulation results well correspond to the measured data and demonstrate that the developed model can precisely predict the rate-dependent hysteresis. We also investigate the parameters' properties with hysteresis characteristics. In the developed model, the length of the minor radius describes the MHE varying with the input frequencies and amplitudes, while the length of major radius and the orientation of the ellipses represent the decreasing PPOA phenomenon. Finally, a real-time feedforward controller with an inverse model is designed to compensate for the rate-dependent hysteresis under different input frequencies. The experimental results show that the hysteresis effects are obviously reduced at both the lower and higher frequencies.



## 1. Introduction

Piezoelectric translator (PZT) actuators are increasingly popular for high-speed and high-accuracy micro- and nanopositioning systems because of its high output force, large bandwidth and fast response time [1–3]. However, the piezoelectric material exhibits nonlinearities such as creep and hysteresis. The creep phenomenon is the drift of the output displacement for a constant applied voltage, which can be characterized by a definite mathematic model [4]. The hysteresis characteristics are the multi-valued nonlinear phenomenon between the applied voltage and the output displacement. The maximum error caused by the hysteresis can be as much as 10–15% of the output amplitude if the actuators operate in the open-loop strategy [5]. On the other hand, the hysteresis characteristics are generally non-differentiable and rate-dependent. It is difficult to predict the hysteresis for precise positioning and tracking control. Although there is an almost linear relationship between the output displacement and input charge, charge control of PZT actuators has not been widely used due to the complicated circuitry of the power driver [6]. Hence, the voltage-driven strategy and the problem of hysteresis compensation still draw significant research interest.

In the last decade, many mathematic hysteresis models such as Bouc-Wen model, Duhem model, Backlash-like model [7], Preisach model (PM) [8,9] and Prandtl-Ishlinskii model (PIM) [10] have been proposed to describe the hysteresis behaviors. Recently, the most popular models are the PM and PIM, which are originally used to predict the rate-independent hysteresis. Unfortunately, the hysteresis of the PZT actuators is a rate-dependent phenomenon, strongly depending on frequencies and amplitudes of the control input. Therefore, the Preisach and Prandtl-Ishlinskii models have to be modified for the applications with high-speed input. In some new researches [11–14], dynamic density functions have been proposed to predict the rate-dependent hysteresis by introducing varying rates of input or output. Cruz-Hernandez and Hayward [15] propose a simple phase control approach, which represents the hysteresis nonlinearity as a phase lag and attempts to cancel it by using a phase lead transfer function. Ru and Sun [16] and Bashash and Jalili [17] develop polynomial-based models to predict the hysteresis effects, but these models work well with low input frequencies. Moreover, advanced control technologies [3,18,19] such as fuzzy control, neural control and adaptive control are also adopted to compensate for the hysteresis. It should be mentioned that, in previous literatures, the amplitude-dependent

* Corresponding author. Tel.: +86 21 34206545; fax: +86 21 34206086.
*E-mail address:* zhulm@sjtu.edu.cn (L. Zhu).

0924-4247/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.sna.2010.09.020

hysteresis has been addressed extensively, while few publications discuss the rate-dependent hysteresis.

In this work, a new mathematical model is developed to describe the rate-dependent hysteresis effects based on an elliptical model. Since the expressions of the model are completely analytical and can be determined only by a set of parameters, this method simplifies the modeling of complicated hysteresis behaviors. To represent the hysteresis characteristics of the PZT actuators, two definitions – maximum hysteresis error (MHE) and the peak-to-peak output amplitude (PPOA) are given. In the proposed model, the length of the minor radius describes the MHE varying with the input frequencies and amplitudes, while the length of major radius and the orientation of the ellipses represent the decreasing PPOA phenomenon. Based on the proposed model, a feedforward controller with the inverse model is designed to compensate for the rate-dependent hysteresis under different input frequencies. The experimental results show that the hysteresis effects are obviously reduced at both the lower and higher frequencies.

## 2. The new hysteresis model

Ellipses are important geometric primitives in the fields of pattern recognition and computer vision [20], and have also been used in magnetization modeling [21]. However, little publications have been found in PZT modeling. In this work, the ellipses are adopted to describe the rate-dependent hysteresis of the PZT actuators. The proposed model is convenient because the expressions of the ellipses are completely analytical and can be determined only by a set of parameters [22].

### 2.1. Mathematical definitions of an ellipse

Suppose that data points $X_i = (x_i, y_i)^T$, $i \in [1, 2 \ldots]$ are given in an ellipse. These points can be described in the parametric form [20]

$$X_i = X_0 + R(\phi)AP(\theta_i) \tag{1}$$

where $X_0 = (x_c,\ y_c)^T$ are the coordinates of the center, $R(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$, $A = diag(a, b)$, $(a > b)$ and $P(\theta_i) = (\cos\theta_i, \sin\theta_i)^T$. $R(\phi)$ is the rotation matrix, where $\phi$ is the orientation of the ellipse between the major axis and the $x$-axis in the 2D plane. $a$ and $b$ are the lengths of the major and minor radii respectively. $P(\theta_i)$ is the parametric matrix with $\theta_i$ varying from 0 to $2\pi$. Based on the definitions, the eccentricity of the ellipse is expressed as

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{2}$$

Therefore, in order to describe the ellipse there are five parameters to be defined: the coordinate center of the ellipse with respect to both the $x$-axis and $y$-axis (denoted by $x_c$ and $y_c$ respectively), the lengths of the major and minor radii (denoted by $a$ and $b$ respectively), and the radian angle of the major axis with respect to the $x$-axis (denoted by $\phi$). It should be mentioned that a one-to-one mapping between the input and output is constructed through an expanded input space by introducing the parameter $\theta$.

### 2.2. Hysteresis modeling

As shown in Fig. 1, hysteresis loops of a PZT actuator are attained under a fixed amplitude and increasing frequencies in the range 0.5–300 Hz with the form of $v(t) = 75 + 15\sin(2\pi ft)$. In this figure, the $x$-axis represents the input voltage, while the $y$-axis is the output displacement of the PZT actuator. The maximum hysteresis error (denoted by MHE) and the peak-to-peak output amplitude

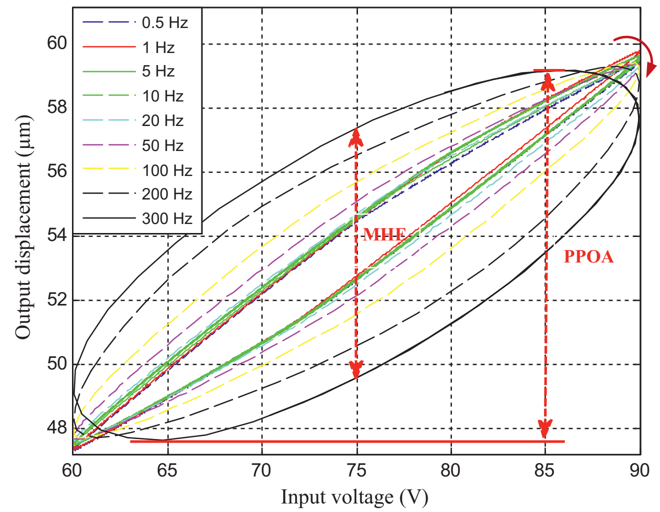


**Fig. 1.** Hysteresis curves with different frequencies of the control input. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

(denoted by PPOA) are used to represent the hysteresis characteristics. Intuitively, the hysteresis effects are more obvious as the frequencies increase. For example, the MHE and PPOA are shown in Fig. 1 with respect to the control input in 300 Hz. Studying the hysteresis curves, it is clear that the MHE becomes larger with increasing frequencies of the control input, while the PPOA decreases. In addition, the hysteresis loops turn clockwise with increasing input frequencies as indicated by the arrow. However, the shape of the hysteresis shows considerable variations at different frequencies. At lower frequencies below 20 Hz, the hysteresis loops have sharp tips, while they become round corners at higher frequencies. This phenomenon is also illustrated in recent experimental reports [14]. Furthermore, the hysteresis curves under a fixed frequency and increasing amplitudes in the form of $v(t) = 75 + A\sin(2\pi 100t)$ are illustrated in Fig. 2. It is obvious that the hysteresis effect is also amplitude-dependent. It should be clarified that, in this work, we mainly focus on the rate-dependent hysteresis behaviors. The hysteresis curves with both major and minor loops are not addressed in the remainder of this paper.

Obviously, hysteresis characteristics of the PZT actuators are rate-dependent, highly depending on the frequencies and ampli-

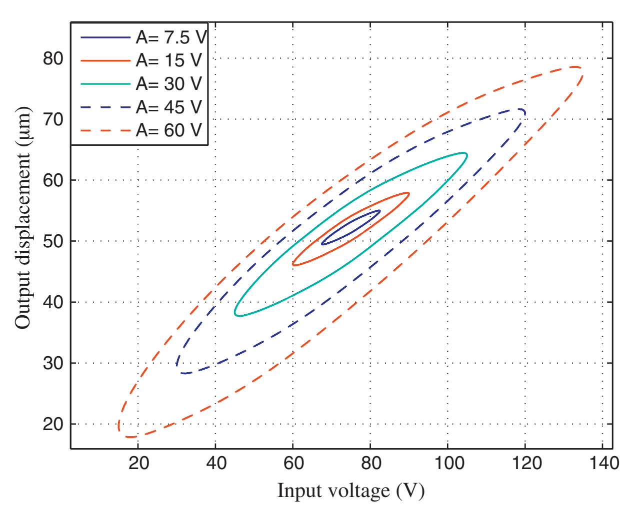


**Fig. 2.** Hysteresis curves with different amplitudes of the control input in 100 Hz. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

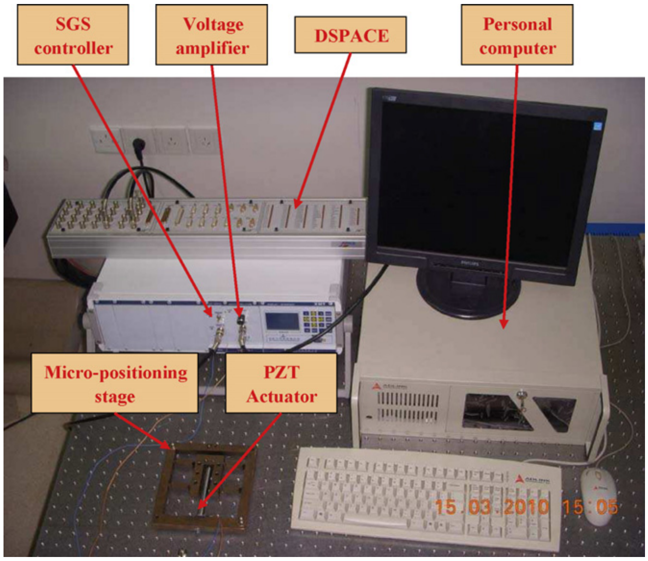


**Fig. 3.** The experimental platform.

tudes of the control input. Also the hysteresis loops are ellipse-like. The distinctions of the hysteresis loops are the MHE and the PPOA of the actuator output. These phenomenons can be directly represented by radii and rotation angles of the ellipses. In this work, the length of the minor radius can describe the MHE, while the length of major radius and the orientation of the ellipses can represent the decreasing PPOA phenomenon. Therefore, the aim of our study is to investigate the influence of rate-dependent hysteresis on the shape of the elliptic models by varying the lengths of the minor and major radii and the orientation of the ellipses. The subsequent job is to extract these parameters from the experiments that have different sets of control input, and investigate the parameters' properties with hysteresis characteristics.

## 3. Experimental setup

In order to construct the hysteresis model, experiments are designed to investigate the influences of the input frequencies on hysteresis characteristics in the PZT actuator. A stack actuator PSt 150/7/100 VS12, which is a preloaded PZT from Piezomechanik in Germany, is used to determine the rate-dependent hysteresis behaviors. The resonance frequency of this actuator is 10 kHz. The actuator provides maximum 100 μm travel and includes an integrated high-resolution strain gauge position sensor (SGS). In this work, the actuator is installed in our designed one-dimension micro-positioning stage with the maximum 90 μm displacement. The experimental platform is shown in Fig. 3. The dSPACE-DS1103 rapid prototyping controller board equipped with 16-bit ADC and 16-bit DAC is adopted to generate and acquire experiment data. The sampling frequency of the system is set as 10 kHz. The excitation signals are amplified by a high-voltage amplifier (HVA) with a fixed gain of 15, which provides excitation voltage for the actuator in the 0–150 V range. Both the input voltage and the feedback displacement are acquired and stored in the computer through the human interface. The direct least square fitting method [23] is used to identify parameters of the elliptic models through the MATLAB offline. Fig. 4 illustrates the structure of the experimental system.

In the experiments, the hysteresis effects were tested under sinusoidal excitation signals under different frequencies sweeping 0.5–300 Hz in the form of $v(t) = 75 + 15\sin(2\pi ft)$. The positive bias of the sinusoidal excitations was used to ensure a positive

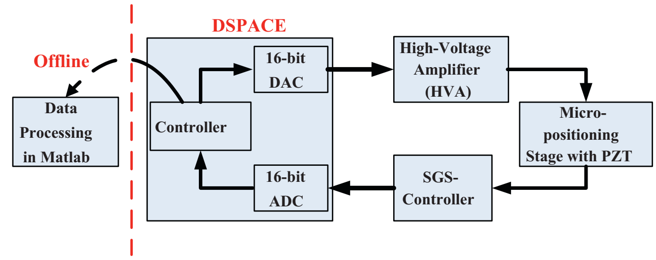


**Fig. 4.** The structure of the experimental system.

excitation of the actuator. The resulting frequency-dependent displacement responses have been shown in Fig. 1 under various excitation frequencies. The results demonstrate that the hysteresis is strongly dependent on the input frequencies, especially at the frequencies above 20 Hz.

## 4. Simulation validation and analysis

Experimental data are applied to identify parameters of the elliptic models using the direct least square fitting method. For the sake of processing the data conveniently, both the input voltage and the output displacement were normalized in the range of [0, 1] with the maximum values (150 V and 90 μm respectively) in hysteresis modeling and controller design. In order to demonstrate the validity of the developed model, the simulation results are compared with the measured data from the actuator at selected frequencies in Fig. 5. The results show that the elliptic model can completely predict the rate-dependent hysteresis of the PZT actuator, especially at higher frequencies. However, due to the nature of the ellipses, the turning point is usually round. This makes some deviations between the developed model and the experimental data when the frequencies are below 20 Hz. This phenomenon is caused by the well known sharp-tip property [24] of the hysteresis loops at lower frequencies. At the other hand, the hysteresis loops become round corner under higher input frequencies, which insures the modeling accuracy using a single ellipse. The evident deviations at lower frequencies are also demonstrated in [14]. In fact, the distortion at lower frequencies can be mitigated by dividing the hysteresis loops into the ascending and descending parts and matching each part with an individual ellipse. It should be mentioned that the deviations can be easily eliminated with a feedback controller, which is being carried out.

Based on the aforementioned simulation, the developed model is efficient to describe the increasing MHE and decreasing PPOA phenomenons as input frequencies increase. As a summary, we present the values of five elliptical parameters calculated from the experimental data under different sinusoidal frequencies in Table 1. Before applying this model to compensate for the rate-dependent hysteresis, the parameters' properties are investigated with the hysteresis characteristics.

**Table 1**
Values of the parameters in the elliptical model under different sinusoidal frequencies.

| Frequency (Hz) | $x_c$ | $y_c$ | $a(f)$ | $b(f)$ | $\phi(f)$ |
|---|---|---|---|---|---|
| 0.5 | 0.4983 | 0.5725 | 0.1129 | 0.0066 | 0.5881 |
| 1 | 0.4980 | 0.5720 | 0.1132 | 0.0061 | 0.5800 |
| 5 | 0.4985 | 0.5729 | 0.1128 | 0.0070 | 0.5775 |
| 10 | 0.4983 | 0.5729 | 0.1135 | 0.0069 | 0.5672 |
| 20 | 0.4982 | 0.5722 | 0.1136 | 0.0080 | 0.5610 |
| 50 | 0.4984 | 0.5728 | 0.1146 | 0.0112 | 0.5516 |
| 100 | 0.4984 | 0.5730 | 0.1150 | 0.0160 | 0.5386 |
| 200 | 0.4987 | 0.5724 | 0.1144 | 0.0258 | 0.5107 |
| 300 | 0.4990 | 0.5720 | 0.1117 | 0.0355 | 0.4912 |

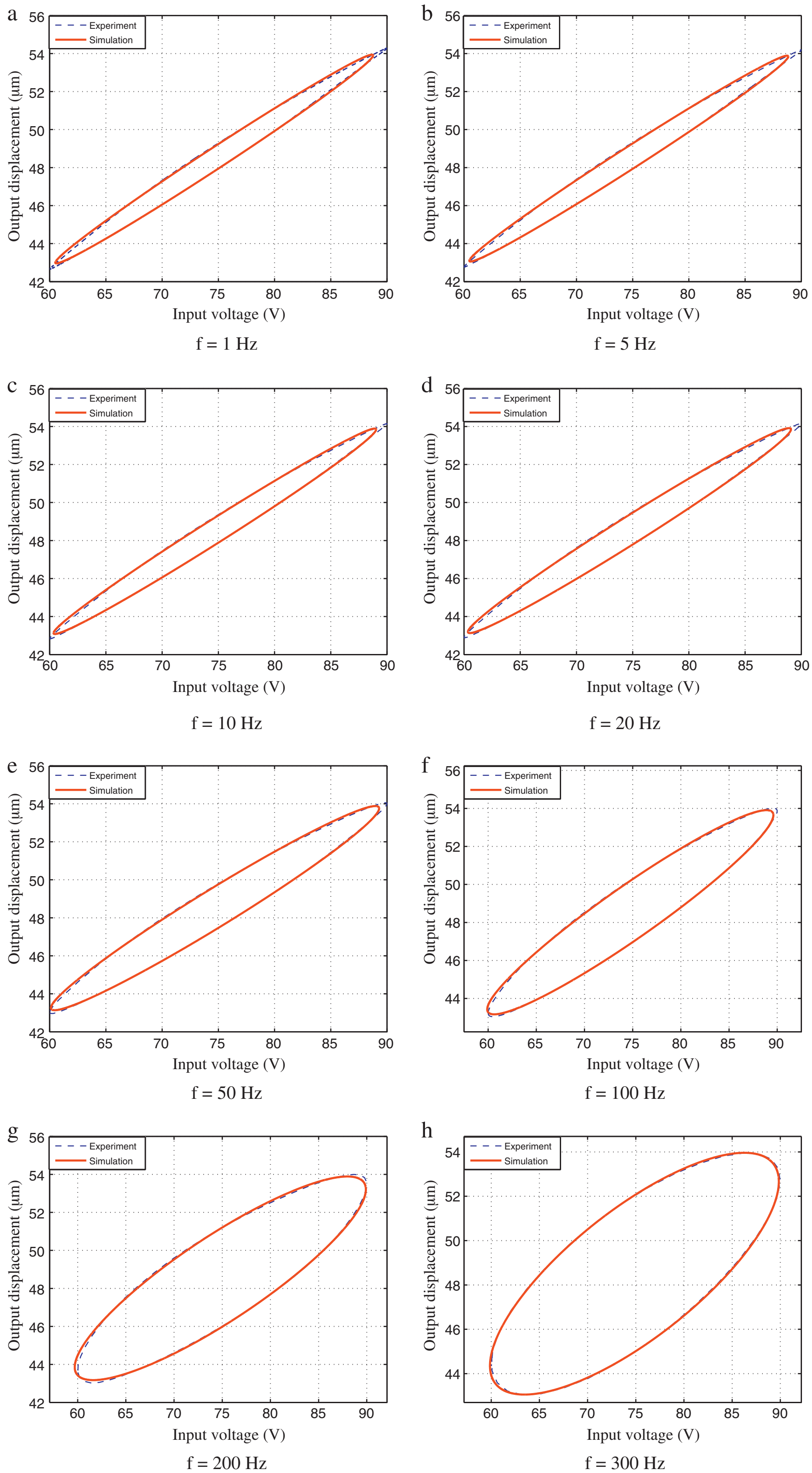


**Fig. 5.** Comparisons of the hysteresis loops between the experimental results and the simulation results predicted by the developed model under sinusoidal excitations at different frequencies (blue dash-experimental data, red solid-simulation results). (a) $f$ = 1 Hz, (b) $f$ = 5 Hz, (c) $f$ = 10 Hz, (d) $f$ = 20 Hz, (e) $f$ = 50 Hz, (f) $f$ = 100 Hz, (g) $f$ = 200 Hz, (h) $f$ = 300 Hz. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

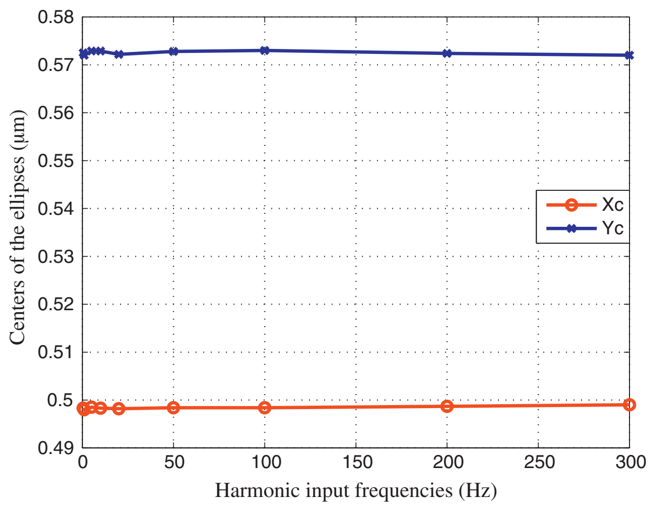


**Fig. 6.** Coordinate centers of the ellipses under different frequencies(red circle-$x_c$; blue star-$y_c$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

### 4.1. Centers of the ellipses

Fig. 6 shows the center values of the model with the input frequencies, where $x_c$ and $y_c$ are the *x*-axis and *y*-axis coordinates of the centers respectively. This figure shows that $y_c$ are almost constant as the frequencies of control input increase. However, the slight difference of $y_c$ is creep-like, which can be explained by the creep effect of the PZT actuator for a constant $x_c$ [4]. Therefore, the hysteresis model can be represented with a family of concentric ellipses, and the number of elliptic parameters is reduced to only three which characterize the rate-dependent hysteresis behaviors.

### 4.2. Length of major and minor radii

Fig. 7 shows the variances of the major-radius and minor-radius length (denoted by $a(f)$ and $b(f)$ respectively) with different frequencies. Because of the sharp tips of the hysteresis loops at lower frequencies below 20 Hz, the length of the major radius becomes a little lager with increasing frequencies. However, the length starts to shorten slightly when the frequencies are above 20 Hz.

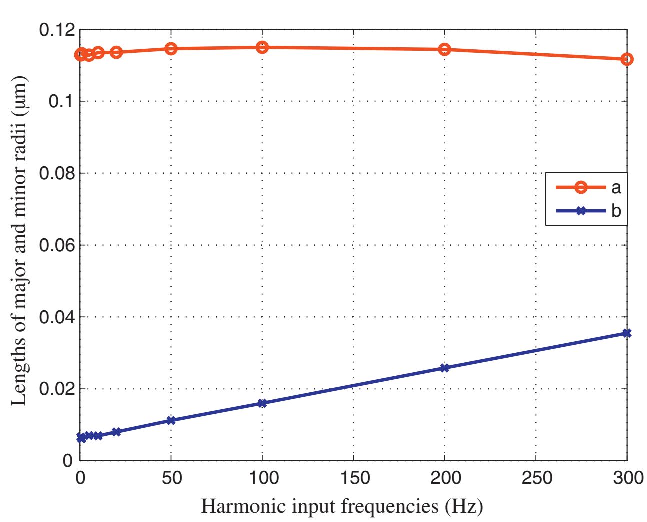


**Fig. 7.** Length of major and minor radii of the developed model under different input frequencies (red circle-major radius $a(f)$; blue star-minor radius $b(f)$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

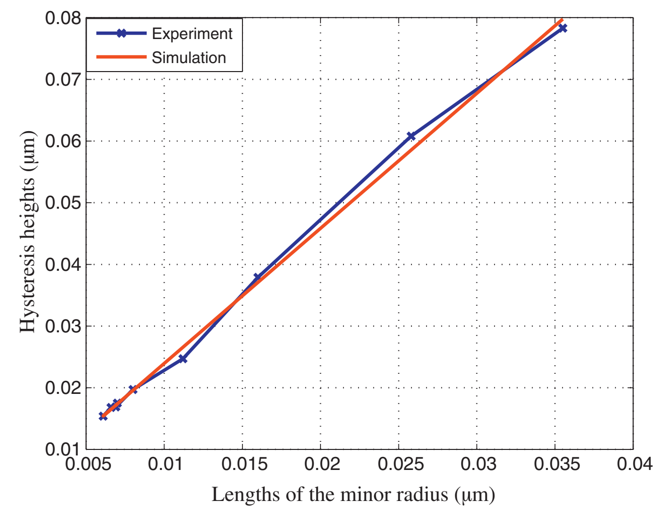


**Fig. 8.** The MHEs vary with the length of the minor radius under different frequencies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

This can interpret that the PPOA begins to decrease as frequencies increase. Also, the length of the minor radius becomes larger with the increasing frequencies. As shown in Fig. 8, the MHEs are approximate linear with the minor-radius length as input frequencies increase. The results demonstrate that the minor radius can well predict the rate-dependent MHE.

### 4.3. The orientation of the elliptic model

Fig. 9 shows the radian angles of the ellipses with different frequencies. It is obvious that the curves of the model turn clockwise as frequencies increase, which well fits with the experimental hysteresis loops. Since the range of the input voltage is constant, this rotation results in decreasing PPOAs. This conclusion agrees well with the discussions in Section 2.2. As shown in Fig. 10, the PPOAs are compared with the radian angles of the ellipses under different frequencies. At the input frequencies below 20 Hz, the PPOAs are linear with the radian angles of the developed model. However, the relation curve between the PPOAs and the radian angles should be described as a quadratic function with the frequencies above 20 Hz.

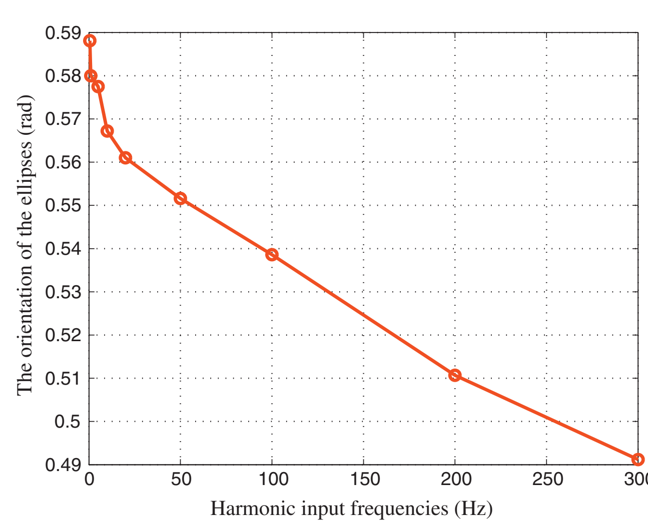


**Fig. 9.** The radian angles of the model under different frequencies.

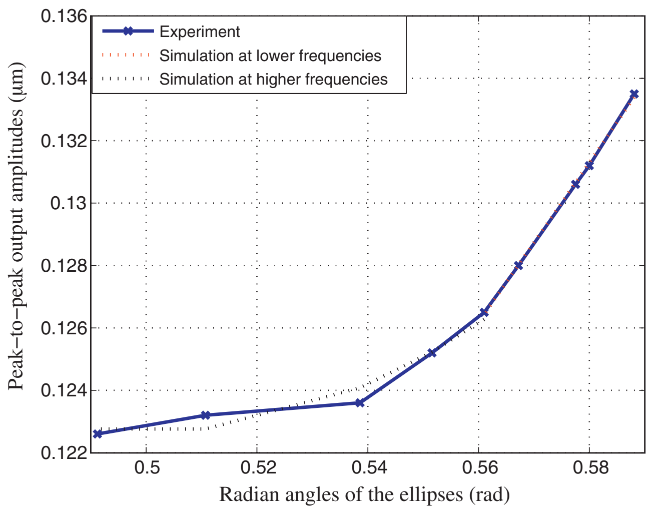


**Fig. 10.** The PPOAs vary with the radian angles of the ellipses under different frequencies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

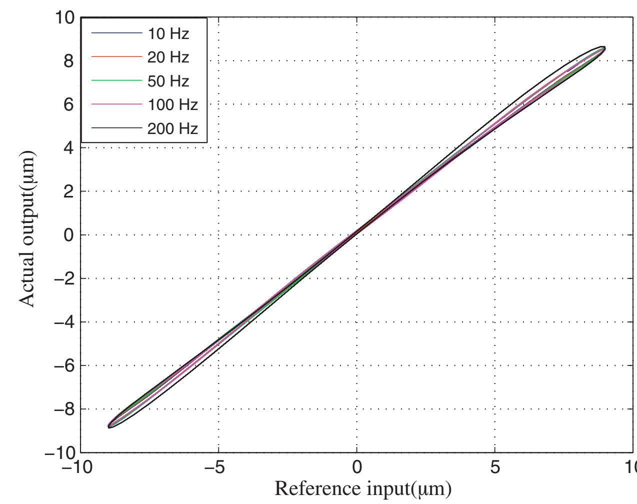


**Fig. 12.** The compensated hysteresis curves with an inverse model at different frequencies. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of the article.)

## 5. Experimental validation result

In order to verify the new developed hysteresis model of PZT actuators, a feedforward controller with an inverse model is developed to track 18 μm peak-to-peak (p–p) sinusoidal trajectories with different frequencies selected as 10 Hz, 20 Hz, 50 Hz, 100 Hz and 200 Hz. The control scheme is shown in Fig. 11.

### 5.1. Controller design

In the hysteresis modeling, the control input voltage $v(t)$ is described by the $x$-axis coordinates of the ellipses, while the output displacement $y(t)$ is the $y$-axis coordinates. According to the aforementioned discussions and equation (1), the following set of equations is derived

$$\begin{cases} v(t) = v_c + r_v(f)\sin(2\pi ft + \alpha_1(f)) \\ y(t) = y_c + r_y(f)\sin(2\pi ft + \alpha_2(f)) \end{cases} \tag{3}$$

where $r_v(f) = \sqrt{a^2(f)\cos^2(\phi(f)) + b^2(f)\sin^2(\phi(f))}$, $r_y(f) = \sqrt{a^2(f)\sin^2(\phi(f)) + b^2(f)\cos^2(\phi(f))}$, $\alpha_1(f) = \arctan\,(b(f)/a(f)\tan\,(\phi(f))), \alpha_2(f) = \arctan\,(-\,b(f)/a(f)\cot\,(\phi(f)))$ and $\alpha_2(f) < \alpha_1(f)$. Therefore, the output displacement of the actuators has the same period with the control input voltage. The only differences are the positive bias, amplitude and phase shift. Since the centers of the family of the ellipses characterizing the hysteresis are constant under different input frequencies as addressed in Section 4.1, the variable bias between the input and output can be described as a rate-independent mapping function. Based on the parameters of the ellipses identified in Section 4, the least square fitting method is used to identify the mapping function. Hence, for a given control input in the form of $v(t) = v_c + r_v\ \sin(2\pi ft)$, the output of the piezoelectric actuators can be gotten as follows

$$y(t) = H_1(v_c)v_c + H_2(r_v, f)r_v\ \sin(2\pi ft) \tag{4}$$

where $H_1(\cdot)$ represents the rate-independent mapping function to describe the variable bias, $H_2(\cdot)$ represents the rate-dependent hysteresis function to describe the variable amplitude and phase shift. In Section 4, we conclude that the hysteresis model can be represented by a family of concentric ellipses with frequency-dependent $a(f)$, $b(f)$, and $\phi(f)$. Hence, the rate-dependent hysteresis in PZT actuators can be considered as a rate-dependent phaser $H_2(\cdot)$ combing a constant bias. The subsequent work focuses on the design of $H_2(\cdot)$ consisting of a frequency-dependent variable gain and a frequency-dependent phase delay.

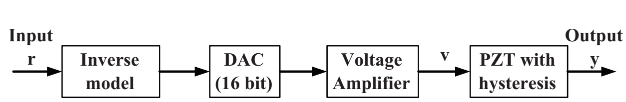


**Fig. 11.** The feedforward controller diagram.

According to (3) and (4), the frequency response of the rate-dependent phaser $H_2(\cdot)$ is derived as

$$H_2(s) = k(r_v, f)\exp(-d(r_v, f)s) \tag{5}$$

where $k(r_v, f) = r_y(f)/r_v(f)$ is the amplitude attenuation, and $d(r_v, f) = \alpha_1(f) - \alpha_2(f)$ is the phase delay. Since the parameters $a(f)$, $b(f)$, and $\phi(f)$ of the elliptic model under different input frequencies have been gotten in Table 1, the least square fitting method is used to identify the parameters involved in the amplitude attenuation function $k(r_v, f)$ and phase delay function $d(r_v, f)$. Then, the inverse phaser model is obtained as

$$H_2^{-1}(s) = (1/k(r_v, f))\exp(d(r_v, f)s). \tag{6}$$

In this work, the inverse phaser is implemented easily in Simulink of MATLAB using the dSPACE controller board.

### 5.2. Test result

The feedforward controller is designed to predict and linearize the rate-dependent hysteresis in piezoelectric actuators based on the developed hysteresis model. It should be noticed that the inverse model-based feedforward controller is lack of robustness and sensitive to the modeling errors and parameter uncertainties. The actual trajectory usually has a little tracking bias comparing with the designed trajectory. Actually, these errors can be eliminated easily by a feedback controller using the actual output trajectory deviations from the reference trajectory. For purpose of comparing the results with different frequencies clearly, the centers of the hysteresis loops are shifted to origin of the coordinates in this work. Fig. 12 shows the resulting hysteresis curves compensated by the feedforward controller under selected frequencies. In the figure, the hysteresis effects are obviously reduced comparing with the uncompensated ones in Fig. 1 at both the lower and high frequencies. To quantify the performance of the feedforward controller, the maximum tracking error (MTE) and the root-mean-square tracking error (RMSTE) under different frequencies are summarized in Table 2. The results demonstrate that the

**Table 2**
Tracking performance of the feedforward controller with different frequencies (values reported by the percentage of the 18 μm p–p displacement range).

| Frequency (Hz) | MTE (%) | RMSTE (%) |
|---|---|---|
| 10 | 2.86 | 1.1 |
| 20 | 2.81 | 1.15 |
| 50 | 2.92 | 1.22 |
| 100 | 3.22 | 1.36 |
| 200 | 3.27 | 1.52 |

inverse model based controller well compensates for the rate-dependent hysteresis.

## 6. Conclusion

In this paper, a new ellipse-like model is proposed to describe the rate-dependent hysteresis in PZT actuators. The designed sinusoidal excitations under different frequencies are used to identify the model through MATLAB offline. The simulation results predicted by the elliptical hysteresis model agree well with the experimental data. An inverse-model-based feedforward controller is also designed to compensate for the rate-dependent hysteresis with the fixed input amplitude under different frequencies. The simulation and experimental results verify the validity of the proposed model. Several distinct features of the work are summarized as follows.

(1) By introducing the elliptical parameter $\theta$, we construct an expanded input space to precisely describe the multi-valued hysteresis function. Hence, there is a one-to-one mapping between the input and output.
(2) In the proposed model, the length of the minor radius describes the MHE varying with the input frequencies and amplitudes, while the length of major radius and the orientation of the ellipses represent the decreasing PPOA phenomenon.
(3) Based on the developed model, we obtain the relation expression between the input voltage and output displacement. Then, a real-time rate-dependent phaser is implemented to compensate for the rate-dependent hysteresis.

On the other hand, there is one limitation of the elliptical model in this work. Actually, the hysteresis in PZT actuators is both frequency-dependent and amplitude-dependent. In other words, the parameters of the developed model are functions of both frequency and amplitude. However, it is difficult to analytically express these parameters as functions of the two variables (amplitude and frequency). In this work, we concentrate the discussions on the frequency-dependent hysteresis assuming that the amplitude is constant. Therefore, the hysteresis effects with both the major and minor loops are not addressed. In addition, the feedforward controller is high sensitivity to modeling errors and parameter uncertainties. Hybrid control strategies combining the feedforward and feedback controllers will be designed to realize more precise positioning and rapid tracking control.

## Acknowledgments

This work was partially supported by the National Natural Science Foundation of China under Grant No. 91023047, the Science and Technology Commission of Shanghai Municipality under Grant Nos. 09520701700 and 09JC1408300.

## References

[1] H. Jung, J.Y. Shim, D. Gweon, Tracking control of piezoelectric actuators, Nanotechnology 12 (1) (2001) 14–20.
[2] S.S. Aphale, S. Devasia, S.O.R. Moheimani, High-bandwidth control of a piezoelectric nanopositioning stage in the presence of plant uncertainties, Nanotechnology 19 (12) (2008) 125503.
[3] U.X. Tan, W.T. Latt, F. Widjaja, C.Y. Shee, C.N. Riviere, W.T. Ang, Tracking control of hysteretic piezoelectric actuator using adaptive rate-dependent controller, Sensors and Actuators A: Physical 150 (1) (2009) 116–123.
[4] H. Jung, D.G. Gweon, Creep characteristics of piezoelectric actuators, Review of Scientific Instruments 71 (4) (2000) 1896–1900.
[5] M.S. Tsai, J.S. Chen, Robust tracking control of a piezoactuator using a new approximate hysteresis model, Journal of Dynamic Systems, Measurement, and Control 125 (1) (2003) 97–102.
[6] S. Devasia, E. Eleftheriou, S.O.R. Moheimani, A survey of control issues in nanopositioning, IEEE Transaction on Control Systems Technology 15 (5) (2007) 802–823.
[7] C.Y. Su, Y. Stepanenko, J. Svoboda, T.P. Leung, Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis, IEEE Transaction on Automatic Control 45 (12) (2000) 2427–2432.
[8] P. Ge, M. Jouaneh, Tracking control of a piezoceramic actuator, IEEE Transaction on Control Systems Technology 4 (3) (1996) 209–216.
[9] G. Song, J.Q. Zhao, X.Q. Zhou, J.A. de Abreu-Garcia, Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model, IEEE/ASME Transactions on Mechatronics 10 (2) (2005) 198–209.
[10] H. Janocha, K. Kuhnen, Real-time compensation of hysteresis and creep in piezoelectric actuators, Sensors and Actuators A-Physical 79 (2) (2000) 83–89.
[11] Y.H. Yu, Z.C. Xiao, N.G. Naganathan, R.V. Dukkipat, Dynamic Preisach modeling of hysteresis for the piezoceramic actuator system, Mechanism and Machine Theory 37 (1) (2002) 75–89.
[12] M. Al Janaideh, C.Y. Su, S. Rakheja, Development of the rate-dependent Prandtl-Ishlinskii model for smart actuators, Smart Materials and Structures 17 (3) (2008) 035026.
[13] U.X. Tan, W.T. Latt, C.Y. Shee, C.N. Riviere, W.T. Ang, Feedforward controller of ill-conditioned hysteresis using singularity-free Prandtl-Ishlinskii model, IEEE/ASME Transactions on Mechatronics 14 (5) (2009) 598–605.
[14] M. Al Janaideh, S. Rakheja, C.Y. Su, Experimental characterization and modeling of rate-dependent hysteresis of a piezoceramic actuator, Mechatronics 19 (5) (2009) 656–670.
[15] J.M. Cruz-Hernandez, V. Hayward, Phase control approach to hysteresis reduction, IEEE Transactions on Control Systems Technology 9 (1) (2001) 17–26.
[16] C.H. Ru, L.N. Sun, Hysteresis and creep compensation for piezoelectric actuator in open-loop operation, Sensors and Actuators A-Physical 122 (1) (2005) 124–130.
[17] S. Bashash, N. Jalili, A polynomial-based linear mapping strategy for feedforward compensation of hysteresis in piezoelectric actuators, Journal of Dynamic Systems, Measurement and Control 130 (3) (2008) 1–10.
[18] R. Shahnazi, N. Pariz, A.V. Kamyad, Adaptive fuzzy output feedback control for a class of uncertain nonlinear systems with unknown backlash-like hysteresis, Communications in Nonlinear Science and Numerical Simulation 15 (8) (2010) 2206–2221.
[19] X.L. Zhang, Y.H. Tan, A hybrid model for rate-dependent hysteresis in piezoelectric actuators, Sensors and Actuators A: Physical 157 (1) (2010) 54–60.
[20] F. Dufrenois, Ellipse fitting with uncertainty and fuzzy decision stage for detection. Application in videomicroscopy, in: Proceedings of 6th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty, 2001, pp. 432–443.
[21] M. Schneider, J. Liszkowski, M. Rahm, W. Wegscheider, D. Weiss, H. Hoffmann, J. Zweck, Magnetization configurations and hysteresis loops of small permalloy ellipses, Journal of Physics D-Applied Physics 36 (18) (2003) 2239–2243.
[22] E. Della Torre, E. Pinzaglia, E. Cardelli, Vector modeling – Part II. Ellipsoidal vector hysteresis model. Numerical application to a 2D case, Physica B-Condensed Matter 372 (1–2) (2006) 115–119.
[23] A. Fitzgibbon, M. Pilu, R.B. Fisher, Direct least square fitting of ellipses, IEEE Transactions on Pattern Analysis and Machine Intelligence 21 (5) (1999) 476–480.
[24] M. Rao, H.R. Krishnamurthy, R. Pandit, Magnetic hysteresis in two model spin systems, Physical Review B 42 (1) (1990) 856–884.

## Biographies

**GuoYing Gu** received B.E. degree (with honors) in electronic engineering from Shanghai Jiao Tong University, China, in 2006. He is currently working toward the Ph.D. degree in mechanical engineering at Shanghai Jiao Tong University. His research interests include mechatronics, motion control, modeling and control of piezoelectric actuators.

**LiMin Zhu** received the B.E. degree (with honors) and the Ph.D. degree in mechanical engineering from Southeast University in 1994 and 1999, respectively. From November 1999 to January 2002, he worked as a postdoctoral research fellow in Huazhong University of Science and Technology. In March 2002, he was appointed associate professor in the Robotics Institute, Shanghai Jiao Tong University. Since August 2005, he has been a professor. He has held the visitorship in Monash University (from September 1997 to May 1998) and The City University of Hong Kong (from December 2000 to March 2001). His research interests include mechatronics, CAD/CAM and mechanical signature analysis.